ÉCOLE POLYTECHNIQUE
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Some Observations on Equation-Based Rate Control

Milan Vojnović

Jointly with Jean-Yves Le Boudec

ICA

Institute for computer
Communications and Applications

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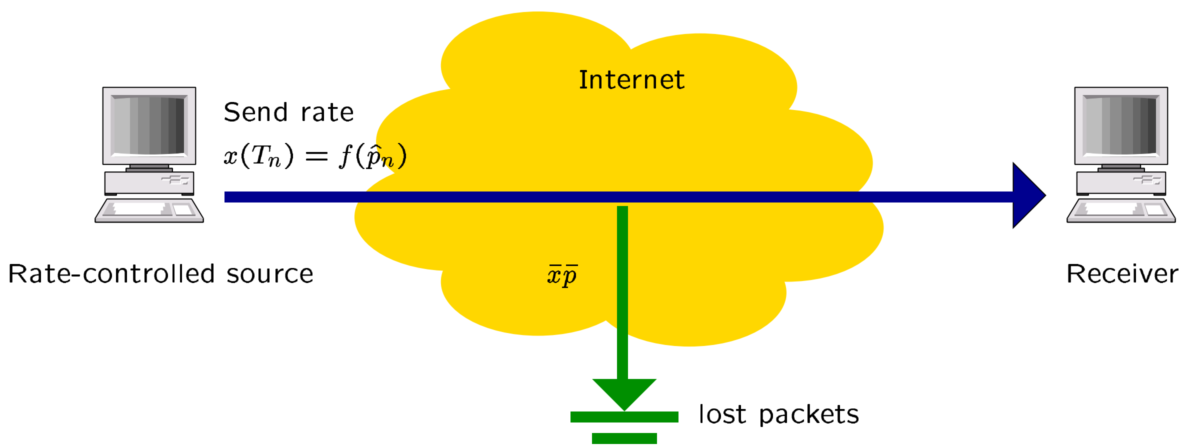
Control System that We Study



f is a loss-throughput formula

$\{T_n\}$ are rate-update instants

$\bar{p}$ is the long-run loss-event ratio

$\hat{p}_n$ is estimator of $\bar{p}$ at T_n

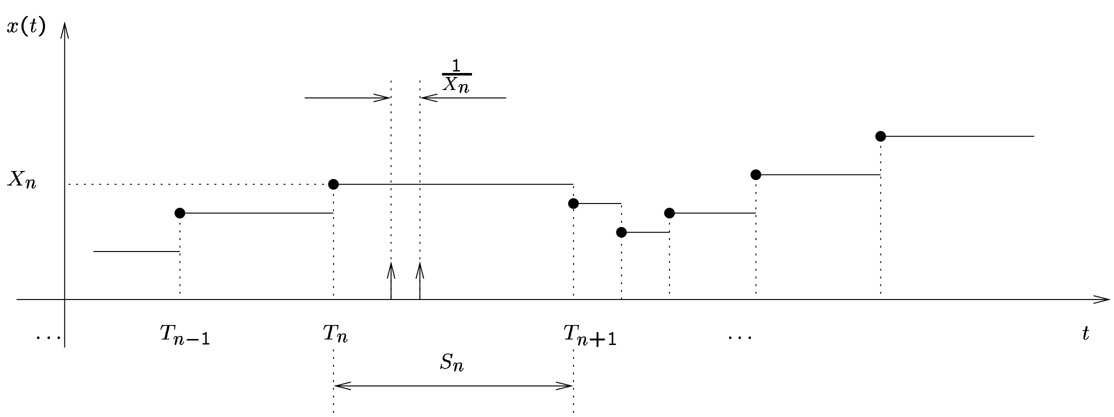
Control System that We Study (cont'd)

Rate controlled as:

$$X_n = f(\hat{p}_n) \qquad (1)$$

where X_n is the sending rate at T_n

and $x(t) = X_n$, $T_n \le t < T_{n+1}$



Why we Study Such a Rate Control?

Such rate controls are proposed for media streaming over the Internet.

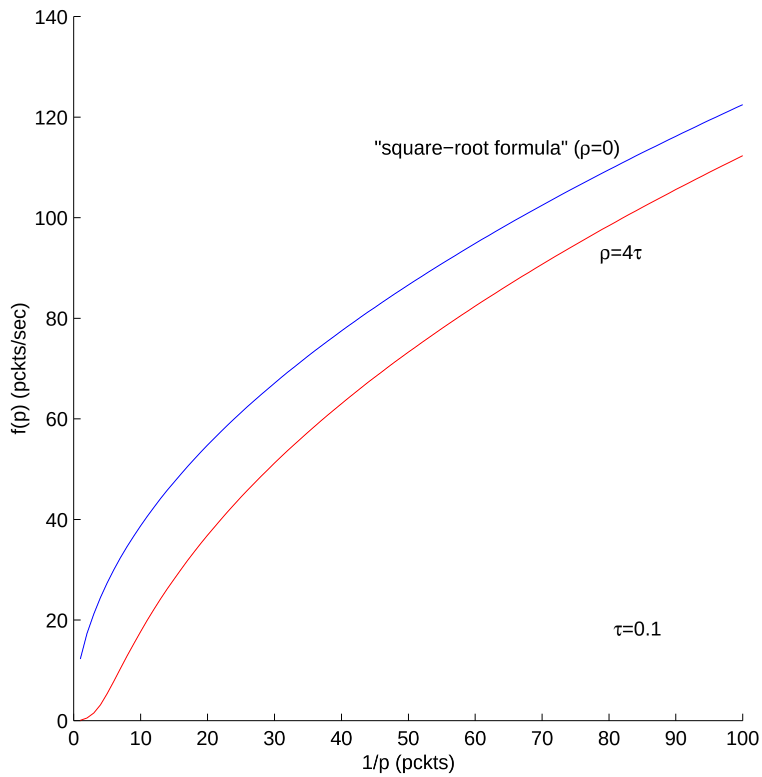
In the Internet, function f relates $\bar{p}$ to the throughput of a TCP source.

In fact, f is also function of some round-trip time statistics
(we focus only on the loss-originating effects)

It is required that the rate control is TCP-friendly.

TCP-friendliness: Under the same operating conditions, the rate control does not achieve higher throughput than a TCP source.

Some Typical Functions f



Padhye at al approximate formula (ToN, 8(2), 2000):

$$f(p) = \frac{1}{\tau a p^{1/2} + \rho b p^{3/2} + \rho c p^{5/2}} \text{ (pcks/sec)}$$

where τ and ρ are round-trip time and TCP retransmission timeout, respectively, and a, b, c positive-valued constants.

Problem

Does it hold

$$\mathbb{E}[x(t)] \leq f(\bar{p}) \text{ ?}$$

If yes, we say the control is conservative.

If the control is conservative, then it is TCP-friendly.

Two Special Assumptions

(A1) $\{T_n\}$ are the loss-event instants

(A2) $1/\widehat{p}_n$ is an unbiased estimator of $1/\bar{p}$

Both assumptions motivated by TFRC proposal (www.aciri.org/tfrc).

With TFRC, $\widehat{p}_n = 1/\widehat{\theta}_n$, where

$$\widehat{\theta}_n = \sum_{l=1}^{L} w_l \theta_{n-l+1}$$

where w_l, $l = 1, \ldots, L$, are some positive numbers summing to unity, and θ_n is the number of the packets sent in $[T_n, T_{n+1})$

Note, for ergodic system: $\bar{p} = 1/\mathbb{E}[\theta_0]$

Thus, $\mathbb{E}[1/\widehat{p}_n] = \mathbb{E}[\widehat{\theta}_n] = \mathbb{E}[\theta_0] = 1/\bar{p}$

$\Rightarrow$ (A2) verified

Some Preliminary Observations

For $f(p)$ concave with $1/p$:

$$\mathbb{E}[X_n] \leq f(\bar{p})$$

- But $\mathbb{E}[x(t)]$ is not the same as $\mathbb{E}[X_n]$
- $\mathbb{E}[X_n]$ is the expected rate at special time points; it is the rate as seen at loss-event instants (Palm expectation)

Some Preliminary Observations (cont'd)

Relation between $\mathbb{E}[x(t)]$ and $\mathbb{E}[X_n]$ depends on the statistics of the point process $\{T_n\}$

By Palm inversion formula:

$$\mathbb{E}[x(t)] = \frac{\mathbb{E}[X_n \sigma(X_n)]}{\mathbb{E}[\sigma(X_n)]}$$

where $\sigma(x) := \mathbb{E}[S_n | X_n = x]$

and $S_n = T_{n+1} - T_n$

Main Result

Theorem 1 *If*

(C1) $f(p)$ *is concave with* $1/p$ *and*

(C2) $\sigma(x)$ *is non-increasing with* x*,*

then

$$\mathbb{E}[x(t)] \leq f(\bar{p})$$

in other words, the control is conservative.

The theorem identifies sufficient conditions under which the control is provably conservative.

Discussion of the Sufficient Condition (C1)
$f(p)$ is concave with $1/p$

- True for some simple functions f

  E.g., the square-root formula

- Not true for small values of $1/p$ with more complex f

  E.g., as seen earlier for Padhye et al formula

Discussion of the Sufficient Condition (C2) $\sigma(x)$ is non-increasing with x

If there exists a hidden congestion state that evolves slowely, then, the expected time between losses given the rate x may become NOT non-increasing with x.

Validation by Modeling

The general model is:

$$X_{n+1} = f(1/\sum_{i=1}^{L} w_l X_{n-l+1} S_{n-l+1})$$

Note: $\theta_n = X_n S_n$

Suppose $\{S_n\}$ is a stationary random process.

Then, the model is an autoregressive process with stationary random coefficients.

Two Special Cases

f is non-linear $\Rightarrow$ the throughput not computed for the general model.

We study two special cases:

Case 1) the square-root formula with $L = 1$

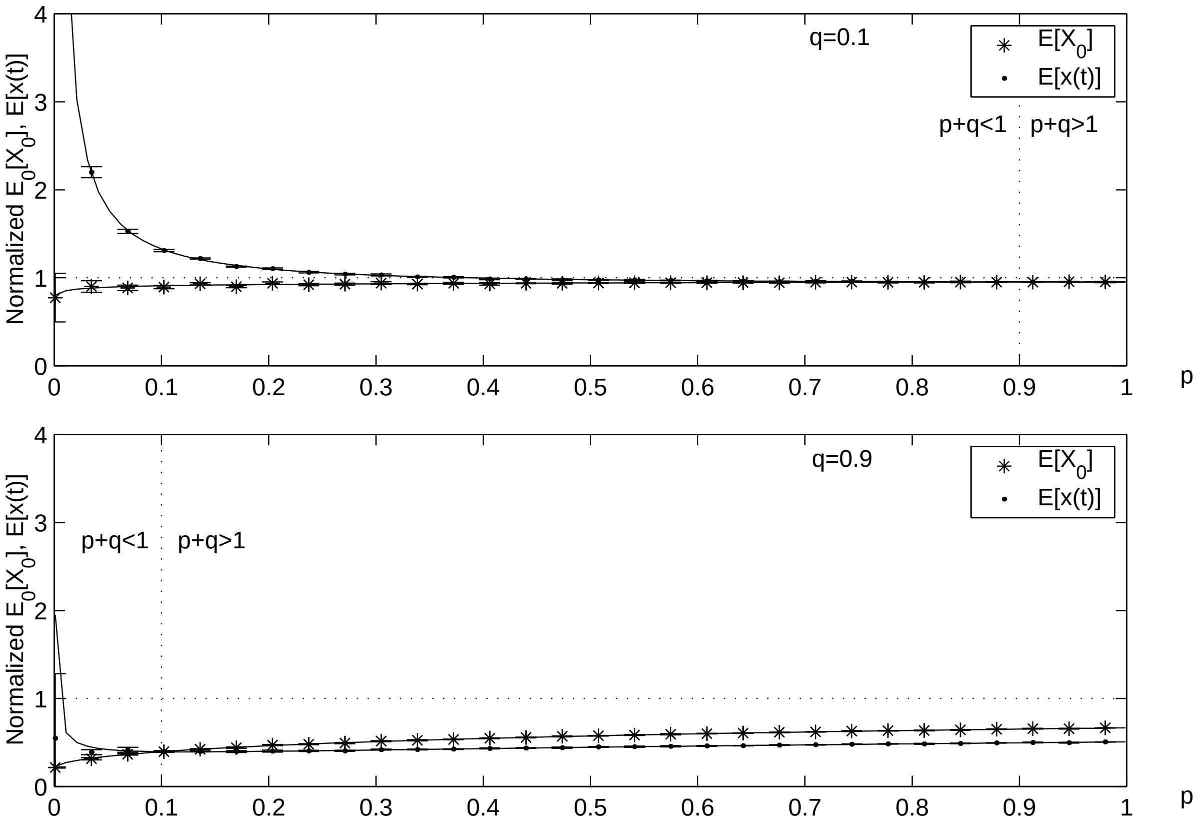
Case 2) the linearized system with $L \geq 1$

We consider a simple case: $\{S_n\}$ governed by a hidden discrete-time Markov chain $\{Z_n\}$.

Details omitted. For a 2-state hidden Markov chain, we compute the throughput numerically for Case 1) and a closed-form expression is retrieved for Case 2).

Some Numerical and Simulation Results

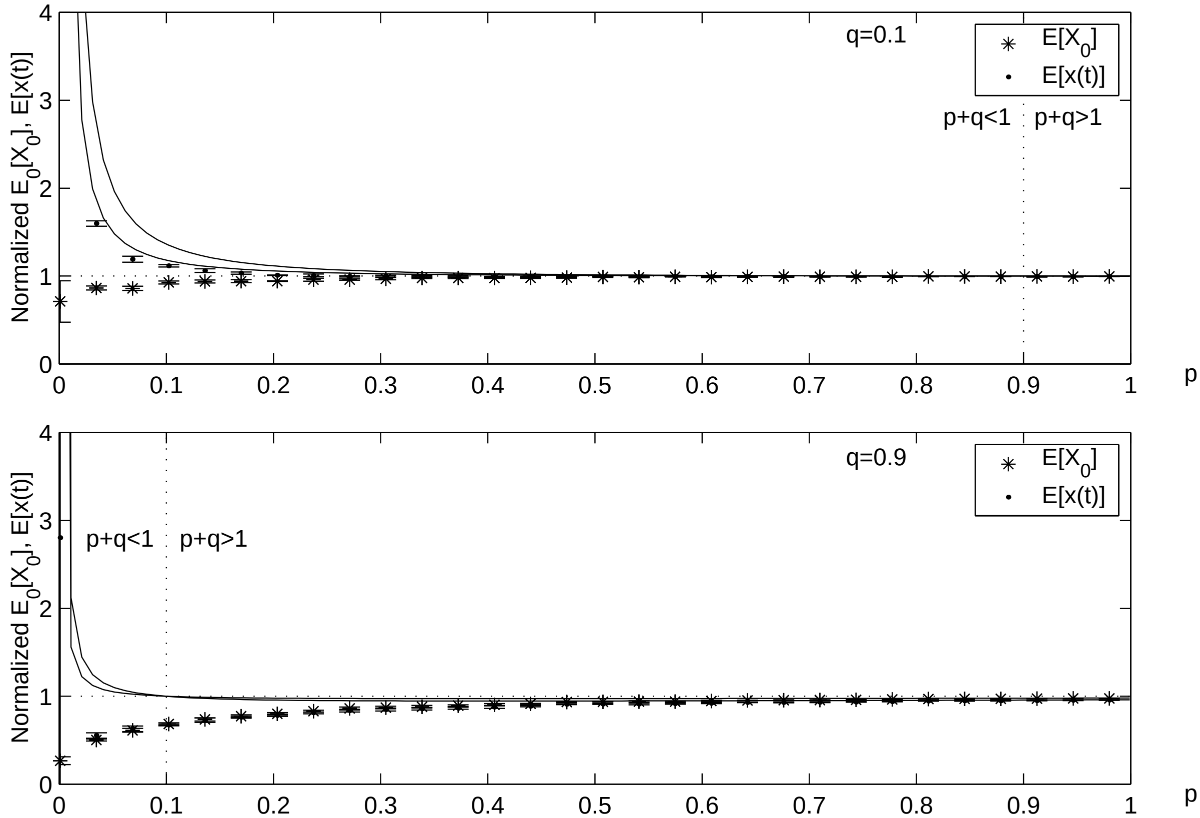
Case 1) the square-root formula with $L = 1$



Note: p and q are transition probabilities of the 2-state hidden Markov chain

Some Numerical and Simulation Results (cont'd)

Case 2) the linearized system with $L \geq 1$



Note: p and q are transition probabilities of the 2-state hidden Markov chain

Discussion of the Results

1) There exists statistics of the loss-event inter-arrival times such that the control is non-conservative

   - Condition (C1), $f(p)$ is concave with $1/p$, is true

   - Condition (C2), $\sigma(x)$ is non-increasing with x, must not be true in the non-conservative regime

2) The non-conservative behavior comes with positively correlated loss-event inter-arrival times (not shown in the slides)

3) The analytical results for the linearized system deviate from the simulations for small q to p ratio (this is explained by increased variance $\text{Var}[S_0] \sim q/p$)

Overly Conservative Nature of the Control

Several empirical studies reported elsewhere indicate: *TFRC is overly conservative as the loss-event ratio gets high.*

We identify a cause of this phenomena.

Overly Conservative Nature of the Control

Consider the f used in TFRC:

$$f(p) = \frac{1}{\tau a p^{1/2} + \rho b p^{3/2} + \rho c p^{5/2}} \text{ (pcks/sec)}$$

where τ and ρ are round-trip time and TCP retransmission timeout, respectively, and a, b, c positive-valued constants.

Overly Conservative Nature of the Control (cont'd)

Consider Bernoulli (q) packet loss model; then

$$\sigma(x) = \frac{1}{qx}$$

And:

$$\begin{aligned}
\mathbb{E}[x(t)] &= \frac{1}{\mathbb{E}[\frac{1}{X_n}]} \\
&= \frac{1}{\mathbb{E}[\frac{1}{f(\hat{p}_n)}]} \\
&= \frac{1}{\tau a \mathbb{E}[\hat{p}_n^{1/2}] + \rho b \mathbb{E}[\hat{p}_n^{3/2}] + \rho c \mathbb{E}[\hat{p}_n^{5/2}]} \\
&\leq \frac{1}{\tau a \bar{p}^{1/2} + \rho b \bar{p}^{3/2} + \rho c \bar{p}^{5/2}}
\end{aligned}$$

Observations:

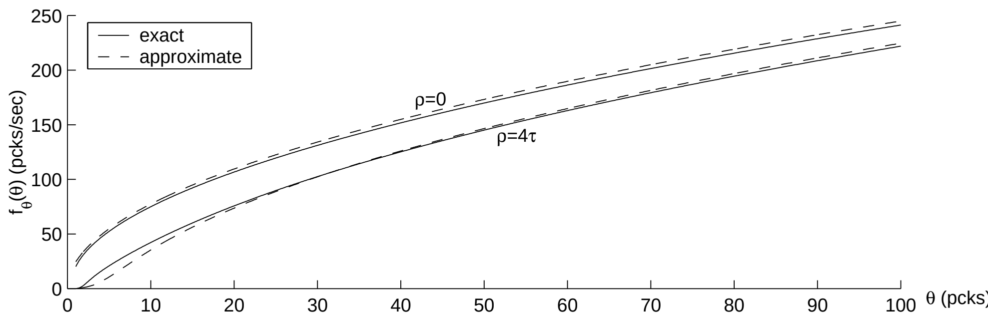
1) $\hat{p}_n^{1/2}$, $\hat{p}_n^{3/2}$ and $\hat{p}_n^{5/2}$ terms are all convex with respect to $1/\hat{p}_n$

2) $\hat{p}_n^{3/2}$ and $\hat{p}_n^{5/2}$ come into play for high loss-event ratio

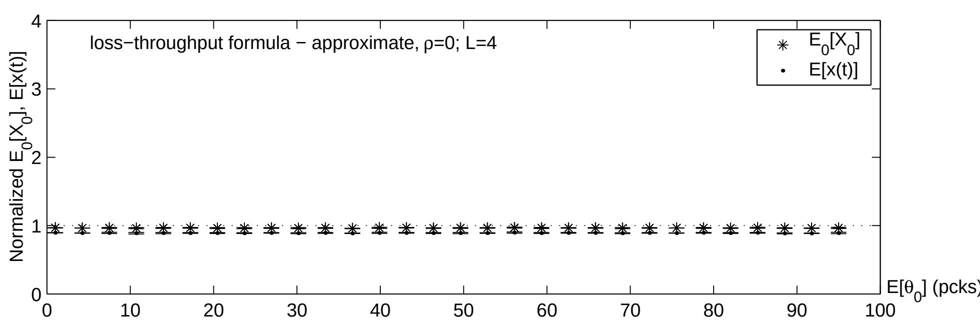
3) they are steep in this region and convexity is resulting in the overly conservative throughput

Overly Conservative Nature of the Control: Numerical Example

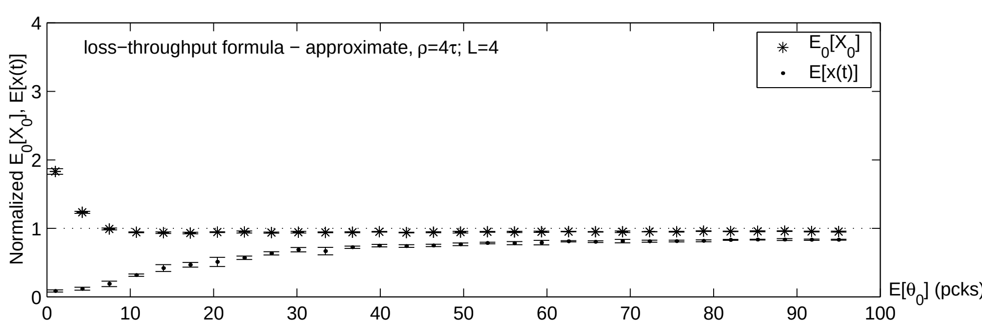
$f(p)$ versus $1/p$



The square-root formula



The approximate Padhye et al formula



<u>Note</u>: With the square-root formula the phenomena does not exist; with the approximate Padhye et al formula, yes.

Two Origins of a Conservative Control

(1) the rate update at loss-event instants

(2) non-linearity of f (concavity)

Note, given that our sufficient conditions hold:

$$\mathbb{E}[x(t)] \overset{(1)}{\leq} \mathbb{E}[f(\widehat{p}_n)] \overset{(2)}{\leq} f(\bar{p})$$

Some Variants of the Control

Some rate controls do not update the rate at the loss-event instants

(e.g., the rate updated upon receiving periodic RTP reports from the receiver)

In such a case, it is reasonable to suppose $\{S_n\}$ be an i.i.d. random process and thus:

$$\mathbb{E}[x(t)] = \mathbb{E}[X_n]$$

We consider two cases ...

Some Variants of the Control: First Case

$\widehat{p}_n$ is unbiased estimator of $\bar{p}$

and $f(p)$ convex with p

Then

$$\mathbb{E}[x(t)] \geq f(\bar{p}) \;(= \text{ iff } \widehat{p}_n \equiv \bar{p})$$

Note: the control is always non-conservative.

Example: $\widehat{p}_n = \sum_{l=1}^{L} 1_{Z_{N(T_n)-l+1}=1}$

where $Z_n = 1$ if the n-th packet is lost,

$Z_n = 0$, otherwise

$N(T_n)$ is the sequence number of the latest packet sent before T_n; for simplicity, the feedback delay ignored

Some Variants of the Control: Second Case

$1/\widehat{p}_n$ is unbiased estimator of $1/\bar{p}$

and $f(p)$ convex with $1/p$

Then

$$\mathbb{E}[x(t)] \geq f(\bar{p}) \ (= \text{ iff } \widehat{p}_n \equiv \bar{p})$$

Note: the control is always non-conservative.

Example: $\widehat{p}_n = \sum_{l=1}^{L} w_l \theta_{n-l+1}$

and $f(p)$ Padhye at al formula for large p

Conclusion

We believe our results would help us in understanding and designing valid rate controls.

In particular, we show:

1) Sufficient conditions ensuring a conservative control.

2) A cause of an overly conservative nature of a TFRC-like control for high loss rate.

How do we eliminate non-linearity effects?

$\Rightarrow$ increase the smoothing of the loss estimator
$\Rightarrow$ diminishes responsiveness $\Rightarrow$ Trade-off

Further Details

M. Vojnović and J.-Y. Le Boudec, "Some Observations on Equation-Based Rate Control", ITC-17, Salvador de Bahia, Brazil, 2001.

On-line at:

http://icawww.epfl.ch/vojnovic